Chapter 5, 1

Consider simple regression

$$y = \beta_0 + \beta_1 x_1 + u$$

under MLR.1 - MLR.4

Show $\text{plim}\,\hat{\beta}_0 = \beta_0$

$$E(y) = \beta_0 + \beta_1 E(x) \Rightarrow \beta_0 = E(y) - \beta_1 E(x)$$

Facts:

- $\text{plim}\,\hat{\beta}_1 = \beta_1$
- continuous mapping theorem]: If $T_n \xrightarrow{p} a$, $U_n \xrightarrow{p} b$ $\Rightarrow T_n U_n \xrightarrow{p} ab$, $T_n + U_n \xrightarrow{p} a+b$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}_1$$

$$\begin{aligned}
\text{plim}\,\hat{\beta}_0 &= \text{plim}(\bar{y} - \hat{\beta}_1 \bar{x}_1) \\
&\overset{CMT1}{=} \text{plim}\,\bar{y} - \text{plim}\,\hat{\beta}_1 \bar{x}_1 \\
&\overset{CMT1}{=} \text{plim}\,\bar{y} - (\text{plim}\,\hat{\beta}_1)(\text{plim}\,\bar{x}_1) \\
&= E(y) - \beta_1 E(x)
\end{aligned}$$

Chapter 5, 2

mutual funds worker can choose from

$$pctstck = \beta_0 + \beta_1 funds + \beta_2 risktol + u$$

Satisfies MLR.1-MLR.4

percent of pension in stock market

risk tolerance

what is inconsistency of $\tilde{\beta}_1$ from regression of pctstck on funds?

5.4: $$\text{plim}\,\tilde{\beta}_1 - \beta_1 = \frac{Cov(\tilde{u}, funds)}{Var(funds)}$$

$$= \frac{Cov(\beta_2 risktol, funds)}{Var(funds)} + \frac{Cov(u, funds)}{Var(funds)}$$

0

$$= \beta_2 \frac{Cov(risktol, funds)}{Var(funds)}$$

$= \beta_2 \delta_1 \leftarrow$ regression of risktol on funds

Chapter 9,1

$$\log(salary) = \beta_0 + \beta_1 \log(sales) + \beta_2 \log(mktval) + \beta_3 profmarg + \beta_4 ceoten + \beta_5 comten + u$$

$R^2 = 0.353,\ n = 177$

when $ceoten^2$ and $comten^2$ included, $R^2 = 0.375$

Is there functional form misspecification?

$$F = \frac{(R^2_{ur} - R^2_r)/q}{(1-R^2_{ur})/(n-k-1)}$$

$$= \frac{(0.375 - 0.353)/2}{(1-0.375)/(177-7-1)}$$

$$\approx 2.97$$

$F^{10\%}_{2,\infty} = 2.3$, $F^{5\%}_{2,\infty} = 3$

Chapter 15,3

Consider simple regression

$$y = \beta_0 + \beta_1 x + u$$

z binary instrument for x

Show $\hat{\beta}_1^{IV} = (\bar{y}_1 - \bar{y}_0)/(\bar{x}_1 - \bar{x}_0)$

$\bar{y}_p, \bar{x}_p$ averages of y and x (resp.) when $z = p$.

$* \bar{z} = \frac{m}{n}$, m observations s.t. $z_i = 1$

$$\hat{\beta}_1^{IV} = \frac{\sum_{i=1}^{n}(z_i-\bar{z})(y_i-\bar{y})\left(\frac{n}{m(n-m)}\right)}{\sum_{i=1}^{n}(z_i-\bar{z})(x_i-\bar{x})\left(\frac{n}{m(n-m)}\right)}$$

$$\begin{aligned}
\sum_{i=1}^{n}(z_i-\bar{z})(y_i-\bar{y}) &= \sum_{i=1}^{n}(z_i-\bar{z})y_i \\
&= \sum_{z_i=1}(1-\bar{z})y_i + \sum_{z_i=0}\bar{z}y_i \\
&= (1-\bar{z})\sum_{z_i=1}y_i - \bar{z}\sum_{z_i=0}y_i \\
&= \frac{n-m}{n}\sum_{z_i=1}y_i - \frac{m}{n}\sum_{z_i=0}y_i \\
&= \frac{n}{m(n-m)}\left[\frac{n-m}{n}\sum_{z_i=1}y_i - \frac{m}{n}\sum_{z_i=0}y_i\right] \\
&= \frac{1}{m}\sum_{z_i=1}y_i - \frac{1}{n-m}\sum_{z_i=0}y_i \\
&= \bar{y}_1 - \bar{y}_0
\end{aligned}$$

do same for denominator

Chapter 15, 11

$$y_t = \beta_0 + \beta_1 x_t^* + u_t$$
$$x_t = x_t^* + e_t \leftarrow \text{CEV}$$

u_t zero mean, uncorrelated with x_t^*, e_t
e_t zero mean, uncorrelated with x_t^*
x_t^* zero mean

i) $x_t^* = x_t - e_t$

$$\Rightarrow y_t = \beta_0 + \beta_1(x_t - e_t) + u_t$$
$$= \beta_0 + \beta_1 x_t + \underbrace{(u_t - \beta_1 e_t)}_{v_t}$$

$$Cov(v_t, x_t) = Cov(u_t - \beta_1 e_t, x_t^* + e_t)$$

$$= \overbrace{Cov(u_t, x_t^*)}^{0} + \overbrace{Cov(u_t, e_t)}^{0} - \beta_1 \overbrace{Cov(e_t, x_t^*)}^{0}$$
$$- \beta_1 Cov(e_t, e_t)$$
$$= -\beta_1 \sigma_e^2 < 0 \quad \text{when } \beta_1 > 0$$

ii) assume u_t, e_t uncorrelated with past values of x_t^*, e_t

$$Cov(x_{t-1}, v_t) = Cov(x_{t-1}^* + e_{t-1}, u_t - \beta_1 e_t)$$
$$= \overbrace{Cov(u_t, x_{t-1}^*)}^{0} + \overbrace{Cov(u_t, e_{t-1})}^{0} - \beta_1 \overbrace{Cov(e_t, x_{t-1}^*)}^{0}$$
$$- \beta_1 Cov(e_t, e_{t-1}) \to 0$$
$$= 0$$

iii) $$Cov(x_t, x_{t-1}) = Cov(x_t^* + e_t, x_{t-1}^* + e_{t-1})$$
$$= Cov(x_t^*, x_{t-1}^*)$$

yes, unless x_t^* represent random shocks

iv) Use x_{t-1} as an Instrument for x_t
lagged Instrument