Chapter 16, #1

$$y_1 = \alpha_1 y_2 + \beta_1 Z_1 + U_1$$
$$y_1 = \alpha_2 y_2 + \beta_2 Z_2 + U_2$$

i) If $\alpha_1 = 0, \alpha_2 \neq 0$

$\Rightarrow$ reduced form: $y_1 = \beta_1 Z_1 + U_1$

Function of Z_1 exogenous, U_1 error

If $\alpha_1 \neq 0, \alpha_2 = 0$

reduced form: $y_1 = \beta_2 Z_2 + U_2$

$$\Rightarrow \beta_2 Z_2 + U_2 = \alpha_1 y_2 + \beta_1 Z_1 + U_1$$

$$\Rightarrow y_2 = (\beta_2/\alpha_1) Z_2 - (\beta_1/\alpha_1) Z_1 + U_2 - U_1$$
$$= \Pi_{22} Z_2 + \Pi_{21} Z_1 + V_2$$

$\alpha_1 \neq 0, \alpha_2 \neq 0$
$\alpha_1 \neq \alpha_2$

ii) multiply second structural equation by α_1/α_2 and subtract from the first

$$y_1 - (\alpha_1/\alpha_2) y_1 = \alpha_1 y_2 - \alpha_1 y_2 + \beta_1 Z_1 - (\alpha_1/\alpha_2)\beta_2 Z_2 + U_1 - (\alpha_1/\alpha_2) U_2$$

$$[1 - (\alpha_1/\alpha_2)] y_1 = \beta_1 Z_1 - (\alpha_1/\alpha_2)\beta_2 Z_2 + U_1 - (\alpha_1/\alpha_2) U_2$$

$$\Rightarrow y_1 = \Pi_{11} Z_1 + \Pi_{12} Z_2 + V_1$$

$$\Pi_{11} = \beta_1 / [1 - (\alpha_1/\alpha_2)], \quad \Pi_{12} = -(\alpha_1/\alpha_2)\beta_2 / [1 - (\alpha_1/\alpha_2)]$$

$$V_1 = [U_1 - (\alpha_1/\alpha_2) U_2] / [1 - (\alpha_1/\alpha_2)]$$

Is $\alpha_1 \neq \alpha_2$ likely to hold in supply and demand models?

reasonable. Suppose y_1 is quantity, y_2 is price and the first equation is a supply model and the second equation is a demand model

then generally $\alpha_1 > 0$, $\alpha_2 < 0$

Chapter 16, 4

Consider SEM

$$\log(earnings) = \beta_0 + \beta_1 alcohol + \beta_2 educ + U$$
$$alcohol = \gamma_0 + \gamma_1 \log(earnings) + \gamma_2 educ + \gamma_3 \log(price) + U_2$$

Suppose $\beta_1, \beta_2, \gamma_1, \gamma_2, \gamma_3$ non-zero which is identified?

the first since log(price) determines alcohol consumption but not log(earnings)
It is a valid instrument and we can estimate by 2SLS.

Chapter 10, #2

$$gGDP_t = \alpha_0 + \delta_0 int_t + \delta_1 int_{t-1} + U_t$$

U_t uncorrelated with $int_t, int_{t-1}, int_{t-2}, \ldots$

Suppose federal reserve policy is

$$int_t = \gamma_0 + \gamma_1 (gGDP_{t-1} - 3) + V_t, \; \gamma_1 > 0$$

(if GDP growth greater than 3%, the Fed increases interest rates to prevent an overheated economy)

If V_t uncorrelated with U_t and $int_t, int_{t-1}, int_{t-2}, \ldots$ argue int_t must be correlated with U_{t-1}

write $gGDP_{t-1} = \alpha_0 + \delta_0 int_{t-1} + \delta_1 int_{t-2} + U_{t-1}$

$$int_t = \gamma_0 + \gamma_1 (\alpha_0 + \delta_0 int_{t-1} + \delta_1 int_{t-2} + U_{t-1} 3) + V_t$$

$$= (\gamma_0 + \gamma_1 \alpha_1 - 3\gamma_1) + \gamma_1 \delta_0 int_{t-1} + \gamma_1 \delta_1 int_{t-2} + \gamma_1 U_{t-1} + V_t$$

$$Cov(int_t, U_{t-1}) = \gamma_1 Cov(U_{t-1}, U_{t-1})$$

$$= \gamma_1 Var(U_{t-1})$$

$$= \gamma_1 \sigma_U^2 \neq 0$$

Violates strict exogeneity TS.2

Chapter 10, #5

quarterly data on: new housing starts
interest rates
real per capita income

model for housing starts that accounts for trends and seasonality

$$\log(nhstart_t) = \alpha_0 + \alpha_1 t + \delta_1 Q2_t + \delta_2 Q3_t + \delta_3 Q4_t + \beta_1 int_t + \beta_2 \log(PCinc_t) + U_t$$